

Math 53 Discussion Problems Nov 21

1. Use Green's Theorem to evaluate the following line integrals.

(a) $\oint_C 3ydx + 2xdy$, where C is the boundary of the region defined by $0 \leq x \leq \pi, 0 \leq y \leq \sin x$, oriented positively

(b) $\oint_C (6y+x)dx + (y+2x)dy$, where C is the circle $(x-2)^2 + (y-3)^2 = 4$, oriented clockwise

(c) $\oint_C \tan^{-1}\left(\frac{y}{x}\right)dx + \ln(x^2 + y^2)dy$, where C is the boundary of the region defined in polar coordinates by $1 \leq r \leq 2, 0 \leq \theta \leq \pi$, oriented positively

2. Use Green's Theorem to find the area under an arch of the cycloid $x = t - \sin t, y = 1 - \cos t$

3. Let C be the boundary of a region R in the xy -plane, oriented positively. Show that the moment of inertia of R about the y -axis equals to $\frac{1}{4} \oint_C x^3 dy - x^2 y dx$